TECHNISCHE UNIVERSITÄT MÜNCHEN

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Soft Matter Theory
Problem Set 1 — Spontaneous Symmetry Breaking in Discrete Systems

hand-out Wed 19.10., return Wed 26.10.

1) Lattice Mean-Field Theory: Consider a system of N interacting classical spins in a magnetic field H at a temperature T, described by the Hamiltonian

$$\mathcal{H}/k_BT = -h\sum_{j=1}^{N} s_j - J\sum_{<i,j>} s_i s_j$$

where each spin can be in three different states $s_j = -1, 0, +1$. Note that $h = H/k_BT$ and $J = K/k_BT$. The sum in the interaction term goes over all nearest neighbor pairs. Each spin has $2D$ nearest neighbors (hypercubic lattice). Use the variational principle with the variational Hamiltonian

$$\mathcal{H}_0/k_BT = -\tilde{h}\sum_{j=1}^{N} s_j$$

and determine the phase transition temperature and the leading term of the magnetization at the phase transition. Compare with the standard two-state Ising model.

2) Transfer Matrix Method: Consider a one-dimensional spin chain of N interacting classical spins in a magnetic field H at temperature T, described by the Hamiltonian

$$\mathcal{H}/k_BT = -h\sum_{j=1}^{N} s_j - J\sum_{j=1}^{N}(s_j+1)^2(s_{j+1}+1)^2$$

where $s_j = -1, +1$. Again $h = H/k_BT$ and $J = K/k_BT$. As in the lecture, we assume periodic boundary conditions, i.e. $s_{N+1} = s_1$.

a) Repeat the steps in the lecture to obtain the Gibbs free energy $G(H,T)$ and from that the magnetization $m =< s_j >$.

b) Determine the entropy per spin S/N, the internal energy and the heat capacity and plot all quantities graphically.

c) Convince yourself that the true free energy calculated by transfer-matrix techniques is indeed lower than the free energy obtained within mean field theory (at least for $T > T_c$).

d) Compare the susceptibilities at high temperatures in the disordered phase obtained with transfer-matrix methods and mean-field theory.