Holographic Quark-Antiquark Potential in Hot, Anisotropic Yang-Mills Plasma

Somdeb Chakraborty

Theory Division
Saha Institute of Nuclear Physics, India

Technical University of Munich
Based on

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S.C. and N. Haque
Exciting times ahead with results from LHC streaming in. Simultaneously, a challenge for theoretical physicists to explain the results.

RHIC, LHC data has provided fascinating insights into properties of QGP.

**QGP behaves like a strongly coupled fluid.**

The AdS/CFT correspondence, or its generalized incarnation - the Gauge/Gravity duality has emerged as a powerful tool to explore such strongly coupled plasma.

*Gravity in the bulk space-time is dual to a gauge theory living on the boundary of the space-time.*
Various alternatives approaches exist to probe dissociation of heavy mesons:

1. Quark-antiquark potential
2. Screening length
3. Decay width of bound states (comes from imaginary-valued potential)

A study of the interaction potential is crucial to our understanding of the bulk properties of QCD plasma phase, e.g. screening property of the QGP, the equation of state, etc.

Has been done in perturbative QCD but results are reliable when coupling is small.

Gauge/gravity duality provides us a handle to probe the strongly coupled QGP.
Why Anisotropy?

- Gauge/gravity duality has so far met with encouraging results in study of hot plasma in equilibrium and isotropic.

  Why Anisotropy?

- Next logical step will be to probe earlier times, \textit{e.g.} before equilibration when the plasma is anisotropic.

- Intermediate stage: \textbf{plasma is anisotropic but in equilibrium.}

- Source of anisotropy \Rightarrow Unequal pressure in longitudinal (beam direction) and transverse directions leading to anisotropic expansion of the plasma.

- We are interested in probing the effects of anisotropy on the quarkonium dissociation.

  Probe it \textit{via} a study of $Q$-$\bar{Q}$ potential and screening lengths
The basic strategy

- Evaluate certain timelike Wilson loops on the gauge theory side which contains information of the $Q$-$\bar{Q}$ potential:

$$\langle W^F(C_{timelike}) \rangle = e^{-VT}$$

$V$: $Q$-$\bar{Q}$ potential as a function of $Q$-$\bar{Q}$ separation $L$
$T$: Length of the loop along the temporal direction.
The superscript $F$ implies it is the Wilson loop in the fundamental representation.
Gauge/gravity duality at play

- Formulation on the gauge theory side is riddled with problems mainly due to the strong coupling involved.
- Ideal setting for invoking the *gauge/gravity duality*
- Set up the suitable gravity dual, solve the problem and come back to the gauge theory side.
- The gravity dual is a certain type IIB solution in string theory with axion and dilaton fields.
- Introduce a probe string whose endpoints are associated with fundamental heavy quark-antiquark pair.
- Calculate the string partition function and identify it with the expectation value of the Wilson loop. [Maldacena, PRL 80 22, 1998]
Gravity Background

We take Mateos-Trancanelli background, [Mateos, Trancanelli, PRL 107 101601, 2012]

\[
ds^2 = r^2 \left( -\mathcal{F}B dt^2 + (dx^1)^2 + (dx^2)^2 + \mathcal{H}(dx^3)^2 + \frac{dr^2}{r^4 \mathcal{F}} \right) + e^{\frac{1}{2} \phi} d\Omega_5^2
\]

\[
\chi = ax^3, \quad \phi = \phi(r)
\]

\[
\mathcal{F}(r) = 1 - \frac{r_h^4}{r^4} + a^2 \mathcal{F}_2(r) + \mathcal{O}(a^4),
\]

\[
\mathcal{B}(r) = 1 + a^2 \mathcal{B}_2(r) + \mathcal{O}(a^4),
\]

\[
\mathcal{H}(r) = e^{-\phi(r)} \quad \text{with} \quad \phi(r) = a^2 \phi_2(r) + \mathcal{O}(a^4)
\]

\[
\mathcal{F}_2(y) = \frac{1}{24r_h^2 y^4} \left[ 8(y^2 - 1) - 10 \log 2 + (3y^4 + 7) \log \left( 1 + \frac{1}{y^2} \right) \right],
\]

\[
\mathcal{B}_2(y) = -\frac{1}{24r_h^2} \left[ \frac{10}{1 + y^2} + \log \left( 1 + \frac{1}{y^2} \right) \right],
\]

\[
\phi_2(y) = -\frac{1}{4r_h^2} \log \left( 1 + \frac{1}{y^2} \right) \quad [y = r/r_h]
\]
\( Q \bar{Q} \)

(a) \hspace{2cm} (b) \hspace{2cm} (c) 

(d) \hspace{2cm} (e)
The string configuration

- The simplest scenario:

  **String lies along** $x^2$ **and moves along** $x^1$ **with velocity** $v$.

- Boost to the rest frame of the string (dipole)

  \[
  dt = \cosh \eta dt' - \sinh \eta dx^1' \\
  dx^1 = -\sinh \eta dt' + \cosh \eta dx^1'
  \]

  where

  \[\eta = \tanh v\]

  is the rapidity. So in this frame the dipole is static while the plasma moves with velocity $v$ in the negative $x^1$ direction.
Final form of the metric

The metric becomes

\[ ds^2 = -A(r)dt^2 - 2B(r)dt dx^1 + C(r)(dx^1)^2 + r^2 ((dx^2)^2 + \mathcal{H}(dx^3)^2) \]
\[ + \frac{dr^2}{r^2 F} + e^{\frac{1}{2} \phi} d\Omega^2_5 \]
\[ \equiv G_{\mu\nu} dx^\mu dx^\nu \]

where

\[ A(y) = (yr_h)^2 \left[ 1 - \frac{\cosh^2 \eta}{y^4} + a^2 \cosh^2 \eta \left\{ F_2 + B_2 \left( 1 - \frac{1}{y^4} \right) \right\} \right], \]
\[ B(y) = (yr_h)^2 \sinh \eta \cosh \eta \left[ \frac{1}{y^4} - a^2 \left\{ F_2 + B_2 \left( 1 - \frac{1}{y^4} \right) \right\} \right], \]
\[ C(y) = (yr_h)^2 \left[ 1 + \frac{\sinh^2 \eta}{y^4} - a^2 \sinh^2 \eta \left\{ F_2 + B_2 \left( 1 - \frac{1}{y^4} \right) \right\} \right]. \]
Nambu-Goto Action

- The string worldsheet action is given by:
  \[ S = \frac{1}{2\pi\alpha'} \int d\sigma d\tau \sqrt{-\text{det}g_{\alpha\beta}} \]

- \( g_{\alpha\beta} \) is the induced metric on the world-sheet:
  \[ g_{\alpha\beta} = G_{\mu\nu} \frac{\partial x^\mu}{\partial \xi^\alpha} \frac{\partial x^\nu}{\partial \xi^\beta} \]

- \( \xi^\alpha = \tau, \sigma \) for \( \alpha = 0, 1 \) respectively.

- String configuration:
  - \( \tau = t, \sigma = x^2, -L/2 \leq x^2 \leq L/2 \)
  - \( r = r(\sigma), \quad x^1(\sigma) = x^3(\sigma) = \text{constant} \)
  - \( r(\sigma) \) is the string embedding satisfying, \( r(\pm\frac{L}{2}) \to \infty \)
Figure: Configuration of the String Probe
If the string moves for a time $T$, the endpoints trace out a rectangle of sides $L$ and $T$ along $x^2$ and $t$ respectively. Identify this rectangle with the relevant Wilson loop on the boundary theory.

Plugging in the form of $g_{\alpha\beta}$, using the above parameterizations

$$S = \frac{T}{2\pi \alpha'} \int_{-L/2}^{L/2} d\sigma \sqrt{A \left(G_{22} + G_{rr}(\partial_\sigma r)^2\right)}.$$ 

where we have scaled $\sigma \to \sigma/r_h$ and $L \to L/r_h$ and $y' = \partial_\sigma y$

The action does not depend explicitly on $\sigma$ leading to

$$L - y' \frac{\partial L}{\partial y'} = \frac{A G_{22}}{\sqrt{A(G_{22} + G_{rr} y'^2)}} = K$$

which, in turn, yields,

$$y' = \frac{1}{K} \sqrt{\frac{G_{22}}{G_{rr}}} \sqrt{AG_{22} - K^2}.$$
\[ L = \frac{2\tilde{K}}{\pi T} \left( 1 + \frac{\tilde{a}^2(5 \log 2 - 2)}{48} \right) \int_{y_t}^{\infty} dy \frac{1}{\sqrt{\left( y^4 - 1 + \frac{\tilde{a}^2}{24} \Sigma \right)}} \times \frac{1}{\sqrt{\left( y^4 - y_c^4 + \frac{\tilde{a}^2}{24} \Lambda(y) \cosh^2 \eta \right)}} \]

\[ \tilde{a} = \frac{a}{r_h} \left( \sim \frac{a}{\pi T} \right), \quad \tilde{K} = K/r_h^2, \quad y_c^4 = \cosh^2 \eta + \tilde{K}^2 \]

\[ \Sigma(y) = 8(y^2 - 1) - 10 \log 2 + (3y^4 + 7) \log \left( 1 + \frac{1}{y^2} \right), \]

\[ \Lambda(y) = 2(1 - y^2) - 10 \log 2 + 2(y^4 + 4) \log \left( 1 + \frac{1}{y^2} \right). \]

\[ y_t = y_c \left( 1 - \frac{\tilde{a}^2}{24y_c^4} \Lambda(y_c) \cosh^2 \eta \right)^{1/4}. \]
Similarly, one finds the action to be

$$ S = \frac{T r_h}{\pi \alpha'} \int_{y_t}^{\infty} dy \sqrt{\frac{y^4 - \cosh^2 \eta + \frac{\tilde{a}^2}{24} \Lambda(y) \cosh^2 \eta}{\left(y^4 - 1 + \frac{\tilde{a}^2}{24} \Sigma(y)\right) \left(y^4 - y_c^4 + \frac{\tilde{a}^2}{24} \Lambda(y) \cosh^2 \eta\right)}} $$

$$ \equiv \frac{T r_h}{\pi \alpha'} \int_{y_t}^{\infty} dy S^{ani}. $$

Plagued by a divergence coming from the self-energy of the $Q-\bar{Q}$ pair. Cured by subtracting the action $S_0$ of two free strings:

$$ S_0 = \frac{T r_h}{\pi \alpha'} \int_{1}^{\infty} dy \sqrt{\frac{y^4 - \cosh^2 \eta + \frac{\tilde{a}^2}{24} \Lambda \cosh^2 \eta}{\left(y^4 - 1 + \frac{\tilde{a}^2}{24} \Sigma\right) \left(y^4 - \cosh^2 \eta + \frac{\tilde{a}^2}{24} \left(\Lambda + \Lambda_{\sqrt{\cosh \eta} \sinh^2 \eta}\right)\right)}} $$

$$ \equiv \frac{T r_h}{\pi \alpha'} \int_{1}^{\infty} dy S^{ani}_0. $$
AdS/CFT Dictionary:

\[ e^{-S_{\text{reg}}} = e^{-(S-2S_0)} = \langle W(C_{\text{timelike}}) \rangle = e^{-V(L)T} \]

\[ r_h \sim \pi T \left[ 1 - a^2 \frac{5 \log 2 - 2}{48 \pi^2 T^2} \right]. \]

\[
\frac{V}{T} = \sqrt{\lambda} \left( 1 - \frac{\tilde{a}^2}{48} (5 \log 2 - 2) \right) \left( \int_{y_t}^{\infty} dy S_{\text{ani}} - \int_{1}^{\infty} dy S_{0\text{ani}} \right)
\]
In the confined phase, the Cornell potential is

\[ V(L) \sim -\frac{\alpha}{L} + \sigma L \]

Coulomb part and confining part.

In the deconfined phase the Coulomb part gets medium modification whereas the string tension \( \sigma \) is usually taken to be zero.

It has been recently shown that it is not justified to take \( \sigma \sim 0 \). Instead there is a medium-dependent contribution from the string tension.

The 2 terms have opposite signs and at large \( L \) the string part dominates.

This is nicely captured in a qualitative way in the plot for \( \eta = 0 \).
Screening Length

- $\eta \to \text{Large}; \quad \tilde{a}^2 \cosh^2 \eta \to \text{Small}$

$$L = \frac{2\tilde{K}}{\pi T} \left(1 + \frac{\tilde{a}^2}{48}(5 \log 2 - 2)\right) \int_{y_t}^{\infty} dy \frac{1}{y^2 \sqrt{y^4 - y_c^4 + \frac{\tilde{a}^2}{24} \Lambda(y) \cosh^2 \eta}} + \ldots$$

Integration yields:

$$L = \frac{2\tilde{K}}{\pi T} \left(1 + \frac{\tilde{a}^2}{48}(5 \log 2 - 2)\right) \frac{\sqrt{\pi}}{y_t^3} \frac{\Gamma(3/4)}{\Gamma(1/4)}$$

$$L_{max} = \frac{1}{\sqrt{\pi T}} \frac{\Gamma(3/4)}{\Gamma(1/4)} \frac{2\sqrt{2}}{3^{3/4}} \frac{1}{\sqrt{\cosh \eta}} \frac{1 - \frac{\tilde{a}^2}{16}(2.9657 \cosh^2 \eta - 0.4885)}{1 - \frac{\tilde{a}^2}{16} \left(\frac{2.9657}{1 - v^2} - 0.4885\right)}$$

- The proportional change brought by anisotropy is,

$$\frac{\Delta L_{max}}{L_{max}|_{\tilde{a}=0}} = -\frac{\tilde{a}^2}{16}(2.9657 \cosh^2 \eta - 0.4885).$$
Figure: $ij$ indicates the dipole moves along $x^i$ and aligns along $x^j$
Figure: \( ij \) indicates the dipole moves along \( x^i \) and aligns along \( x^j \)
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Figure: $ij$ indicates the dipole moves along $x^i$ and aligns along $x^j$
Presence of anisotropy increases the dissociation rate of heavy quarkonium.

Mesons moving along the anisotropic direction are most affected.

Comparison with other models

• In qualitative agreement with results from non-commutative YM theory.
• Contradict results obtained from the Hard Thermal Loop approach of perturbative QCD.