Muonic hydrogen at finite temperature. A toy model for Heavy Quarkonium dissociation

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Based on the work done with Joan Soto (in preparation).
Motivation

Why do we study Muonic Hydrogen (and normal Hydrogen) at finite temperature?

- $J/\psi$ supression in QGP is not fully understood in QCD.
- QED bound states are very similar to heavy quarkonium.
- It may be possible to measure.
Heavy Quarkonium dissociation

- The Coulomb like potential will turn into a Yukawa like potential due to medium effects.
- This dissociation has been observed in different experiments, however there are some experimental data that can not be explained with this naive model.
- The dissociation temperature is still unknown.
Analogy with QED bound states

- At $T = 0$ the same techniques (EFT’s) that are used to find corrections in the QCD static potential can be use to find the Lamb Shift.
- If perturbation theory is used a QGP is not so different to a photon and electrons/positrons plasma.
- It is difficult to obtain information from experiments where the strong interactions is important due to confinement.
- Maybe it is easier with a QED analog experiment.
Analogy with Muonic Hydrogen

Dissociation is mainly due to this diagram.

\[ \begin{array}{c}
\text{In Hydrogen atom this is only relevant at } T \sim m_e, \text{ but as soon as this mechanism is active Hydrogen dissociates. In HQ you can always have gluon loops.}

\text{In Muonic Hydrogen } m_\mu \gg 1/r \sim m_e. \end{array} \]
Possible measurable effects?

- Nowadays, using high intensity lasers one can create electron and positron pairs. [(Gahn et al. Phys. of Plasmas 9, 987 (2002)) and more recently (Chen et al, Phys. Rev. Lett 102, 105001 (2009))].
Muonic Hydrogen in two different situations

Muonic Hydrogen with $m_e = 0$.
- Unrealistic.
- Closer to HQ.

Muonic Hydrogen with actual value of $m_e$.
- The real state one finds in nature.
- Could be measured.
- Not so close to HQ.
Difference between low and high excited states

- For the case $m_e = 0$ there is no difference in the scaling $1/r \gg E$.
- The introduction of a new scale $m_e$ makes it different.
  - For low excited states ($n \leq 2$) $1/r \sim m_e \gg E$.
  - For high excited states ($n \geq 3$) $m_e \gg 1/r \gg E$.

The study of states with different $n$ maybe important experimentally. For example, in the measurements made with Hydrogen atom in the 80’s they use high $n$ Rydberg atoms in order to obtain the same effect with a smaller temperature.
Outline

1. Introduction
2. Review of Hydrogen atom results
3. Muonic Hydrogen with $m_e = 0$
4. Muonic Hydrogen with actual $m_e$
5. Results
6. Conclusions
Quantum field theory at finite temperature

\[ \langle A \rangle_\beta = \frac{1}{\log Z} \text{Tr}(e^{-H\beta} A) \]

\( \beta = 1/T \) is like an imaginary time. [M. Le Bellac, "Thermal Field Theory", Cambridge University Press (1996)]
Quantum field theory at finite temperature
Imaginary-time formalism

\[ \langle \phi(-it)\phi(-it') \rangle_\beta \]

Then analytic continuation.
Real-time formalism

Vertical lines decouple, the two horizontal lines gives a doubling of degrees of freedom.
We can freely choose the longitude of the first vertical line $\sigma$. Actually, we use $\sigma = 0$ (closed-path formalism).
Example: Finite T scalar propagator

\[ \Lambda(K) = \begin{pmatrix} \Lambda_{11} & \Lambda_{12} \\ \Lambda_{21} & \Lambda_{22} \end{pmatrix} \]

\[ \Lambda_{11} = \frac{i}{K^2 - m^2 + i\epsilon} + 2\pi\delta(K^2 - m^2)n_B(|k_0|) \]

\[ \Lambda_{12} = 2\pi\delta(K^2 - m^2)(\Theta(-k_0) + n_B(|k_0|)) \]

\[ \Lambda_{21}(k_0) = \Lambda_{12}(-k_0) \]

\[ \Lambda_{22} = (\Lambda_{11})^* \]
Effective Field Theories are very useful when you have a problem with different scales. In Muonic Hydrogen at finite temperature we have a lot of them.

- Muon mass, $m_\mu$, hard scale.
- Typical radius and trimomentum, $m_\mu \alpha$, soft scale. The electron mass is of this scale.
- Binding energy, $m_\mu \alpha^2$, ultrasoft scale.
- Temperature $T$.
- also $eT$ and $e^2 T$. 
This is the EFT one obtains after integrating out $m_\mu$. 

$$
\mathcal{L} = \psi^+ \left( iD^0 + \frac{\bar{D}^2}{2m_\mu} + \frac{\bar{D}^4}{8m^3_\mu} + c_F e \frac{\bar{\sigma} \bar{B}}{2m_\mu} + c_D e \frac{\bar{\nabla} \bar{E}}{8m^2_\mu} + i c_S e \frac{\bar{\sigma} \bar{D} \times \bar{E} - \bar{E} \times \bar{D}}{8m^2_\mu} \right) \psi + \\
+ N^+ iD^0 N - \frac{1}{4} d_1 F_{\mu\nu} F^{\mu\nu} + \frac{d_2}{m^2_\mu} F_{\mu\nu} D^2 F^{\mu\nu}
$$

This is the EFT one obtains if the $1/r$ scale is also integrated out.

\[ L_{pNRQED} = \int d^3x \left( \bar{\psi} \{ iD^0 + \frac{\vec{D}^2}{2m_\mu} + \frac{\vec{D}^4}{8m_\mu^3} \} \psi + N^+ iD^0 N - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right) + \]
\[ + \int d^3x_1 d^3x_2 N^+ N(t, x_2) \left( \frac{Z\alpha}{|x_1 - x_2|} + \frac{Ze^2}{m_\mu^2} \left( - \frac{cD}{8} + 4d_2 \right) \delta^3(x_1 - x_2) + icS \frac{Z\alpha}{4m_\mu^2} \vec{\sigma} \left( \frac{\vec{x}_1 - \vec{x}_2}{|x_1 - x_2|^3} \times \vec{\nabla} \right) \right) \bar{\psi} \psi(t, x_1) \]

With this lagrangian we can easily get the fine-structure shift and the Lamb shift (first correction due to quantization of electromagnetic field, of order $m\alpha^5 \log(E/m)$). [A. Pineda and J. Soto, Nucl. Phys. Proc. Suppl. 64, 428 (1998)]
Hard Thermal Loops

In a gauge theory at finite temperature.

If $T \gg p$, by power counting $\Pi(p, T) \sim e^2 T^2$. What happens if $p \leq eT$? A resummation is needed (M. LeBellac, Thermal Field Theory), or in the EFT language one needs to integrate out $T$ scale.

$$\mathcal{L}_{YM, HTL} = \mathcal{L}_{YM} + \frac{1}{2} m_D^2 \int \frac{d\Omega_v}{4\pi} \text{Tr} \left[ \left( \frac{1}{\nu D} v^\alpha F_{\alpha\mu} \right) \left( \frac{1}{\nu D} v^\beta F_{\beta}^\mu \right) \right]$$
A single Hydrogen atom put in a thermal bath of photons (and electrons and positrons if the temperature is high enough).

- What is its dissociation temperature?
- What is the mechanism for this dissociation? (Is it the same one that is proposed for Heavy Quarkonium?).

What we found was

- The dissociation temperature for the \( n = 1 \) state is \( T_d \sim 60\text{KeV} \).
- The mechanism turned out to be the appearance of an imaginary part in the potential due to the interaction of the potential photon with electrons and positrons from the thermal bath. This is consistent with QCD studies made recently in perturbation theory although historically the mechanism was thought to be screening.
Hydrogen atom at different temperatures

- \( m_e \gg T \) the effect can be summarized in
  - A mass redefinition of order \( \delta m \sim \frac{\alpha T^2}{m_e} \).
  - A correction of order \( m\alpha^5 \log(T/E) \) that for \( T \gg E \) will cancel the Bethe-Log.

- For \( m_e \sim T \) the main contribution is a modification of the potential.

\[
V(r) = -\frac{\alpha e^{-m_D r}}{r} + \frac{i 16\alpha^2 g(m_e/T) T^3}{\pi m_D^2} \phi(m_D r)
\]

For \( m_e \gg T \) this goes exponentially to the Coulomb potential.
Muonic Hydrogen with $m_e = 0$

- Although unrealistic, similar to Heavy Quarkonium.
- Debye screening and other effects due to vacuum polarization will be active at every temperature.
- For $1/r \gg T$ temperature effects will be very subleading.
- For $1/r \sim T$ there will be a non-trivial modification of the potential (to our knowledge the QCD analog has not been computed yet).
- For $T \gg 1/r$ we see the dissociation of Muonic Hydrogen. Also in this regime the difference between Heavy Quarkonium and Muonic Hydrogen is just modifications of some constants.
The starting point can be pNRQED at $T = 0$. If one wants a precision of $m_\mu \alpha^5$ there is no need to include vacuum polarization effects, hence we are at the same situation as in Hydrogen atom.

There is a correction of the energy of the order of the Lamb shift $(m_\mu \alpha^5)$ that is a non-trivial function of the temperature and it also appears a thermal width of the same size. [Further details in Escobedo and Soto, Phys. Rev. A 78,032520]
One can integrate out $T$ as an intermediate step. 
$pNRQED \rightarrow pNRQED_T$.
This leads to a modification of $pNRQED$ potential.

\[
\delta V_T^{(LO)} = \frac{\alpha \pi T^2}{3m_\mu} + \frac{2\alpha}{3\pi} r^i (E - H)^3 \left( \log \left( \frac{2\pi T}{|E - H|} \right) r_i - \gamma \right) - i \frac{2\alpha T}{3} r^i (E - H)^2 r_i
\]
Part of this computation was already done for Heavy Quarkonium [Brambilla et al. Phys. Rev. D78, 014017 (2008)].

\[
\delta V_T^{(NLO)} = -\frac{3\alpha}{2\pi} \zeta(3) T m_D^2 r^2 + \frac{i\alpha T m_D^2}{6} r^2 \left( \frac{1}{\epsilon} + \gamma + \log \pi - \log \frac{T^2}{\mu^2} + \frac{2}{3} - 4 \log(2) - 2 \frac{\zeta'(2)}{\zeta(2)} \right) + \frac{i\alpha m_D^2 r^i(E-H)r_i}{6} \left( \frac{1}{\epsilon} + 2 \log \left( \frac{\mu}{T} \right) + A \right)
\]
In order to proceed further we have to fix the relation between $eT$ and $E$. For $E \gg eT$, after integrating out the scale $T$ we also integrate out the scale $E$.

\[
\delta E^E_n = -\frac{7\alpha \pi T m_D^2}{24} \langle r^2 \rangle_n
\]

\[
\delta \Gamma^E_n = \frac{\alpha T m_D^2}{3} \langle r^2 \rangle_n \left( \frac{1}{\epsilon} - 2 \log \frac{|E - H|}{\mu} + \frac{13}{6} - 3 \log 2 - \gamma + \log(4\pi) \right)
\]

This would be exactly the same for Heavy Quarkonium [Work in progress in collaboration with Munich Group and Joan Soto]
\[ T \gg eT \gg E \]

Now Hard Thermal Loops enter in the game. This introduces a new energy scale, the Debye mass \( m_D \sim eT \).

But, one can make an expansion in \( (E - H)/m_D \).

\[
\delta E_n^{eTLO} = \frac{\alpha m_D^3}{6} \langle r^2 \rangle_n
\]

\[
\delta \Gamma_n^{eTLO} = \frac{\alpha T m_D^2}{3} \langle r^2 \rangle_n \left( \frac{1}{\epsilon} - \gamma + \log \pi + \log \frac{\mu^2}{m_D^2} + \frac{5}{3} \right)
\]
\[ \delta \Gamma_n^{eT NLO} = -1.64486 \frac{\alpha T m_D}{m_\mu} - \]
\[ - \frac{\alpha m_D^2}{m_\mu} \left( \frac{1}{\epsilon} - 0.524626 - \gamma + \frac{5}{3} + \log(4\pi) + 2 \log \left( \frac{\mu}{m_D} \right) \right) \]
Conclusions for $1/r \gg T$

- Very tiny effects for this range of temperatures.
- Using EFT’s we are close to a precision of $m_\mu \alpha^5$ in the energy states.
- The computation in QCD would be very similar.
In this case the starting point is NRQED at $T = 0$.
One has to integrate out $1/r$ and $T$ at the same time.
The resulting EFT would be pNRQED with a $T$-dependent potential and HTL in the photon and electron sector.
To our knowledge the analogous situation has not been studied in QCD. This computations tell us about the light quark sector.
Potential

\[ \delta V = -\frac{\alpha m_D^2 r}{4} - \frac{3\alpha}{2\pi} \zeta(3) T m_D^2 r^2 - \]
\[ - \frac{3\alpha m_D^2}{2\pi^2 T^2 r} \int_0^\infty \frac{du}{u(e^u+1)} (1 - \cos(2 Tr) - 2 Tr Si(2 Tr)) + \]
\[ + \frac{\alpha m_D^2}{4\pi^2 T^2 r} \int_0^\infty \frac{du}{u(e^u+1)} (2 - 12 T^2 r^2 u^2 + (4 T^2 r^2 u^2 - 2) \cos(2 Tr) + \]
\[ + 2 Tr \sin(2 Tr) + 8 T^3 r^3 u^3 Si(2 Tr)) + \]
\[ + \frac{i\alpha m_D^2 Tr^2}{6} \left( \frac{1}{\epsilon} + \gamma + \log \pi + \log(r \mu)^2 - 1 \right) + \]
\[ - \frac{i3\alpha m_D^2}{2\pi^2 T} \left( \frac{3}{2} \gamma - \log(rT) + \frac{3}{2} - \frac{3}{2} \log \pi + 3 \log 2 \right) + \]
\[ + \frac{i24\alpha m_D^2}{\pi^2 T^2 r} \int_0^\infty \frac{du}{u^4} \sin(Tr)(Li_2(-e^{u/2}) + \frac{u}{2} \log(1 + e^{u/2}) + \frac{\pi^2}{12} - \frac{u^2}{16}) , \]

where

\[ Si(x) = \int_0^x \frac{\sin t}{t} \, dt \]
Comment on the potential

- For $Tr \to 0$, one recovers the previous section contribution from scale $T$.  
- For $Tr \to \infty$ it can not be considered a perturbation.  
- One can also integrate out $eT$ (if $1/r \sim T$ then $eT \gg E$), but the results would be exactly the same from previous section.  
- It has infrared divergences that cancel out with the ultraviolet divergences of integrating out $eT$.  
- We were unable to get an analytic expression. But one can give a value of the potential for each value of $r$ and also compute the energy correction given a wave function.  
- In Heavy Quarkonium this would be the contribution from light quarks, we expect the gluon contribution to be qualitatively similar.
The starting point can be NRQED at $T = 0$. First integrate out $T$ scale:
- HTL in electron and photon sector
- NRQED for the muons gets a $T$-dependent mass redefinition and a correction of $c_D$ Wilson coefficient (related with the Darwin term in atomic physics).
\( eT \sim 1/r \)

Integrate out \( eT \) and \( 1/r \) at the same time.

\[
V(r, T) = -\frac{\alpha e^{-m_D r}}{r} - \alpha m_D + i\alpha T \phi(m_D r)
\]

with

\[
\phi(x) = 2 \int_0^\infty \frac{dzz}{(z^2 + 1)^2} \left[ \frac{\sin(zx)}{zx} - 1 \right]
\]

Exactly the same functional form in HQ and almost the same in Hydrogen atom.

Dissociation temperature is \( T_d \sim m_\mu \alpha^{2/3} / (\log \alpha)^{1/3} \).
\[ eT \gg 1/r \]

- \( T \gg T_D \), Hence no bound state.
The actual system that we find in nature.
Can tell us about the role of charm mass in bottomonium.
We introduce another scale into the problem.
For lower energy states \((n = 1, 2)\) \(m_e \sim 1/r\).
For higher energy states \((n \geq 3)\) \(m_e \gg 1/r\).
The thermal bath does not have electrons nor positrons.
The only thermalized particle is the photon.
In this situation the only difference between normal Hydrogen and Muonic Hydrogen is the mass of its particles.
1/r ∼ T

Note also that 1/r ∼ m_e.

- Mass dependent HTL (quoted in our Hydrogen atom paper).
- Mass dependent potential.

\[ \delta V = -\frac{4\alpha^2 f(m_e \beta)m_e^2 r}{\pi} - \frac{2\alpha^2}{\pi r} \int_0^\infty \frac{du}{\sqrt{u^2+1}(e^{\beta m_e \sqrt{u^2+1}}+1)} (1 - \cos(2m_e ru) - 2m_e ru Si(2m_e ru)) + \]
\[ + \frac{\alpha^2}{3\pi r} \int_0^\infty \frac{du \sqrt{u^2+1}}{u^2(e^{\beta m_e \sqrt{u^2+1}}+1)} (2 - 12m_e^2 r^2 u^2 + (4m_e^2 r^2 u^2 - 2) \cos(2m_e ru) + \]
\[+2m_e ru \sin(2m_e ru) + 8m_e^2 r^3 u^3 Si(2m_e ru)) + \]
\[ + \frac{i4\alpha^2 T^3 g(m_e \beta) r^2}{3\pi} (\frac{1}{\epsilon} + \gamma + \log \pi + \log(r\mu)^2 - 1) - \]
\[ - \frac{i4\alpha^2 T}{\pi (e^{\beta m_e+1})} (-\frac{\gamma}{2} - \log(rT)) - \log \pi + (e^{\beta m_e+1}) \int_0^\infty \frac{du}{u(e^{\beta m_e \sqrt{u^2+1}}+1)} - \int_0^\infty \frac{du \epsilon^{-\beta m_e u}}{u} + \]
\[ + \frac{i32\alpha^2}{\pi r} \int_0^\infty \frac{du}{u^2} \sin(T ru) (Li_2(-e^u \sqrt{1/4+(\beta m_e)^2/u^2}) + u \sqrt{\frac{1}{4} + \frac{(\beta m_e)^2}{u^2}} \log(1 + e^{u \sqrt{1/4+(\beta m_e)^2/u^2}}) + \]
\[ + \frac{\pi^2}{6} - \frac{u^2}{8} - \frac{(\beta m_e)^2}{2} - g(m_e \beta) + \frac{u^2}{8(e^{\beta m_e+1})}) + \]
\[ + \frac{i64\alpha^2 T}{(2\pi)^2} \int_0^\infty \frac{du}{u^3} \left( \frac{\text{Sinc}(m_e ru) - 1}{e^{\beta m_e \sqrt{1+u^2/4}}+1} - \frac{\text{Sinc}(m_e ru) - e^{-\beta^3 m_e^3 u^3}}{e^{\beta m_e+1}} + \frac{\beta m_e u^2}{8} \frac{\text{Sinc}(m_e ru) - e^{-\beta m_e u}}{e^{\beta m_e+1}} \right)(1 - \frac{1}{e^{\beta m_e+1}}) + \]
\[ + \frac{i4\alpha^2 m_e^2 T r^2}{3\pi^2 (e^{\beta m_e+1})} (\frac{1}{\epsilon} - 1 + \gamma + 2 \log(r\mu) + \log \pi) - \]
\[ - \frac{i16\alpha^2 m_e^2}{3\pi^2 T (e^{\beta m_e+1})} \Gamma(-2/3) + \frac{i2\alpha^2 m_e}{\pi^2 (e^{\beta m_e+1})} (1 - \frac{1}{e^{\beta m_e+1}})(1 + \log(rT)), \]
1/r \sim T, definitions

\[ \text{sinc}(x) = \frac{\sin(x)}{x} \]

\[ L_{i2}(x) = \sum_{k=1}^{\infty} \frac{x^k}{k^2} \]
Infrared divergences

There are two infrared divergences.
The first does not go to 0 as $m_e \to 0$.
  
  - Similar to the one found at $m_e = 0$.
  - Scattering of an electron or positron of the thermal bath with a low energy photon that was going from the muon to the proton.

The other one does go to 0 as $m_e \to 0$.

  - It does not have counterpart in $m_e = 0$.
  - Interaction of a low energy photon travelling from the muon to the proton with non-relativistic electrons or positrons from the thermal bath.
Subtle point in the computation

Problem

- The effect of non-relativistic electrons (positrons) seems to enter only through loops.
- But if this is so, the corrections would be pretty suppressed. But if it is suppressed it can not cancel the infrared divergence.

Solution

- The solution lies in the HTL resummation.
- In non-relativistic system the energy scale is much smaller than the typical momentum.
- In resumming HTL it seems correct to use the relation $T \sim m_e \gg p \gg p_0$ (but it is not).
- The result of doing so is different from doing the resummation with $T \sim m_e \gg p, p_0$ and letting the computation "tell" if $p_0$ is small (for example, with a dirac delta).
- The difference between the two methods cancels precisely the new infrared divergence.
Taking what is explained in the previous slides into account, the result is.

\[
\delta E_{en}^T = \frac{\alpha m_D^3}{6} \langle r^2 \rangle_n
\]

\[
\delta \Gamma_{en}^T = \frac{16 \alpha^2 T^3 \langle r^2 \rangle_n}{3\pi} \left( \frac{1}{\epsilon} - \gamma + \log(\pi) + \log \left( \frac{\mu^2}{m_D^2} \right) + \frac{5}{3} \right) +
\]

\[
+ \frac{8 \alpha^2 m_e^2 T \langle r^2 \rangle_n}{3\pi^2 (e^{m_e/T} + 1)} \left( \frac{1}{\epsilon} - 2 \log \left( \frac{m_D}{\mu} \right) - \frac{5}{3} + \log(4\pi) - \gamma - 2 \log(2) \right)
\]
$T \gg 1/r$

- Qualitatively identical to the $m_e = 0$ case.
- Quantitative small corrections suppressed by $m_e/T$. 
Highly excited states \((n \geq 3)\)

- For this states \(m_e \gg 1/r\).
- If \(m_e \gg T\), basically there would not be a qualitative difference between a normal Hydrogen atom.
- If \(m_e \sim T\). First integrate out \(T\) and \(m_e\) to get mass dependent HTL propagators (quoted in Hydrogen atom paper), and after integrate out \(1/r\).
- If \(T \gg m_e\) what is said in the previous slide also applies
The real part of the potential and the imaginary part of the potential are of the same magnitude when

\[ p \sim m_d = (16\alpha)^{1/3} [g(m_E/T)]^{1/3} T \]

The typical momentum transfer is

\[ p \sim \frac{m_\mu \alpha^2}{n^2} \]

We approximate the dissociation temperature with the temperature in which this two scales are the same
Table: Dissociation temperature for the lower lying states of muonic hydrogen

<table>
<thead>
<tr>
<th>n</th>
<th>$T_d$ (MeV)</th>
<th>$m_D$ (MeV)</th>
<th>$m_d$ (MeV)</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>1.7</td>
<td>0.3</td>
<td>0.77</td>
</tr>
<tr>
<td>2</td>
<td>0.46</td>
<td>0.074</td>
<td>0.19</td>
</tr>
<tr>
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<td>0.24</td>
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<td>0.086</td>
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<tr>
<td>4</td>
<td>0.17</td>
<td>0.017</td>
<td>0.048</td>
</tr>
<tr>
<td>5</td>
<td>0.13</td>
<td>0.010</td>
<td>0.031</td>
</tr>
</tbody>
</table>
1S state

Figure: $E$ vs $T$ for the 1S state
1S state. Comments

- The previous plot was done using the results from $1/r \sim T$ section.
- The correction to the energy was computed using

$$\delta E_{1S} = \langle 1S | \delta V | 1S \rangle$$

- The previous formula is not correct for all the temperatures we have plotted, for high enough temperature $\delta V$ is not longer a perturbation.
- However, when this happens one is almost at the dissociation temperature.
$K_{\alpha}$ transition

Figure: $K_{\alpha}$ transition vs $T$
Application to bottomonium. The effect of finite charm mass

<table>
<thead>
<tr>
<th>$m_c$ (MeV)</th>
<th>$T_d$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\infty$</td>
<td>480</td>
</tr>
<tr>
<td>1200</td>
<td>440</td>
</tr>
<tr>
<td>0</td>
<td>420</td>
</tr>
</tbody>
</table>

This was computed with the same techniques as the Muonic Hydrogen dissociation.

The effect of the running of the coupling constant was taken into account.

Setting $m_c = 0$ the error is roughly of the 5 percent.

The effect is important although in principle $m_c$ is bigger than the dissociation temperature.
Conclusions

- The EFT techniques have been proven useful to deal with a non-trivial system with a lot of different energy scales.
- The Muonic Hydrogen have a lot of common characteristics with Heavy Quarkonium.
- An experimental program for Muonic Hydrogen at finite temperature (if technically possible) could also be useful for Heavy Quarkonium dissociation.
- In any quantitative computation on Bottomonium at finite temperature, Charm mass effects must be included.