The Jet Quenching Parameter and Effective Theories

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Outline

1 Introduction
   - The quark-gluon plasma
   - Jet quenching

2 The effective field theory approach

3 An effective theory for the jet
   - Soft-Collinear Effective Theory
   - The Glauber mode
   - Gauge invariance

4 Effective field theories for the medium
   - Electrostatic QCD
   - Relation to the Wilson loop
   - Perturbative calculations

5 Conclusions

Based on 1208.4253 and work in progress
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What is what?

- What is jet quenching?
  → Modification of jet observables due to presence of a thermal medium (e.g. quark-gluon plasma)

- What is a jet?
  → A narrow cone of hadrons with a large energy and a small invariant mass (light hadrons)

- What is the quark-gluon plasma (QGP)?
  → A phase of the strongly interacting matter
The Quark-Gluon Plasma

Properties of QCD at zero temperature

- Confinement: Only color neutral objects observable
  \[\rightarrow \text{Hadrons in the final state}\]
- Dynamical chiral symmetry breaking: Non-vanishing vacuum expectation value of the quark condensate \(\langle \bar{\psi} \psi \rangle\)
  \[\rightarrow \text{Generates nucleon masses even for } m_q = 0\]

- However, things change for increasing \textbf{temperature} \(T\) and baryochemical potential \(\mu\) (baryon density)
The Quark-Gluon Plasma

- The QCD **phase diagram** (qualitative)

![Phase Diagram](image.png)

- Lattice simulations: $T_c \approx 175 \text{ MeV}$
- Use **jet quenching** to learn more about the QCD phases
Production of a Quark-Gluon Plasma

- Relativistic heavy ion colliders, e.g.
  - RHIC with $\sqrt{s} = 200 \text{ GeV per nucleon}$
  - LHC with $\sqrt{s} = 2760 \text{ GeV per nucleon} \text{ (since 2010)}$

- These probe the $\mu \approx 0$ region of the phase diagram, since at high velocities the plasma predominantly consists of created $q\bar{q}$-pairs and only $\sim 5\%$ valence quarks

\begin{itemize}
  \item Lorentz contracted ions
  \item Interpenetration time $0.25 \text{ fm/c}$; thermalizes
  \item Plasma formation time $\sim 1 \text{ fm/c}$ for high enough energy
  \item QGP lifetime $10 \text{ fm/c}$ for LHC
\end{itemize}
Jets in the Quark-Gluon Plasma

- Jets have a clear experimental signature
- They are produced by hard interactions before the formation of the plasma
  \[ \Rightarrow \text{Production calculable at } T = 0 \]

- Subsequently propagate through the plasma
  \[ \Rightarrow \text{By comparison with jets in p-p collisions the properties of the QGP can be analyzed} \]
Plasma Effects on the Jet

Two types of interaction

- **Radiative energy loss** through medium induced gluon radiation (radiated gluons are again subject to in medium interactions)
- **Jet broadening** without energy loss, i.e. change of momentum perpendicular to initial jet direction through interaction with medium constituents
  → Both effects are relevant to the so-called *jet quenching*

- For two jets in p-p collisions one expects them to be back-to-back
- In heavy ion collisions on the other hand, one of the jets can be significantly suppressed due to interactions with the QGP
Experimental Results

- Jet quenching has been observed at PHENIX, STAR (RHIC) and ATLAS, CMS (LHC)

CMS collaboration

- Measurable quantity of interest: Nuclear modification factor

\[ R_{AA} = \frac{d\sigma_{AA}(p_T, y)/dp_T dy}{\langle \sigma_{NN} T_{AA} \rangle d\sigma_{pp}(p_T, y)/dp_T dy} \]

Ratio of observed hadrons in heavy ion collisions to p-p collisions normalized to number of nucleon binary collisions
The Jet Quenching Parameter

There are several approaches to calculate the effect of the medium on jet fragmentation due to Baier, Dokshitzer, Peigne, Schiff, Zakharov, Armesto, Salgado, Wiedemann, Gyulassy, Levai, Vitev, Guo, Wang, Arnold, Moore, Yaffe, . . .

One way to parameterize effect of the medium in these approaches is to introduce a jet quenching parameter

It corresponds to the change of the momentum perpendicular to the original direction of the jet parton per distance traveled

\[ \hat{q} = \frac{\langle k_{\perp}^2 \rangle}{L} = \frac{1}{L} \int \frac{d^2 k_{\perp}}{(2\pi)^2} k_{\perp}^2 P(k_{\perp}) \]

\( P(k_{\perp}) \) is probability to acquire a perpendicular momentum \( k_{\perp} \) after travelling through a medium with length \( L \)

When describing the broadening of the \( k_{\perp} \)-distribution while travelling a distance through the medium by a diffusion equation, \( \hat{q} \) is related to the diffusion constant
The jet quenching parameter $\hat{q}$

- Does not include collinear radiation which changes the energy of the parton (fragmentation)
- Assume that the final virtuality is determined through medium interactions and not the initial hard process

Goals

Find field theoretic definition of $\hat{q}$
Systematic calculation of the contributions to $\hat{q}$
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The relevant scales

- Several scales appear in the process, most notably
  - The energy of the jet $Q$
  - The scale of the medium (temperature) $T$

**Thermal** scales, such as the Debye mass $m_D \sim g T$
the chromomagnetic mass $g_E \sim g^2 T$ (magnetostatic screening)

- In the weak coupling limit ($g$ small) these scales are ordered by their size

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**Approach**

Introduce a series **effective field theories** to

- transparently derive factorization
- obtain a systematic expansion in terms of ratios of scales
- resum possible large logarithms of ratios of scales
The effective field theory approach

- The appropriate field theories are
- full (perturbative) QCD for hard interactions at the scale $Q$ (creation of the primary jet particle)
- **Soft-Collinear Effective Theory** (SCET) for the description of a jet interacting with soft particles at the scale $T$
  
  Bauer et al. ’01; Beneke at al. ’02
- **Electrostatic QCD** (EQCD) for interactions in a thermalized medium where the scale $T$ has been integrated out
  
  Braaten ’95
- **Magnetostatic QCD** (MQCD) for interactions at the non-perturbative scale $g^2 T$
  
  Braaten ’95
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Soft-Collinear Effective Theory

- Small dimensionless ratio $\lambda = \frac{T}{Q} \ll 1$
- Classify modes by the scaling of their momentum components in the different light-cone directions $(n, \bar{n})$
  
  \begin{align*}
  (p^+, p^-, p_\perp) &= (Q, Q, Q) \sim (1, 1, 1) \text{ is called hard} \\
  (p^+, p^-, p_\perp) &= (T, T, T) \sim (\lambda, \lambda, \lambda) \text{ is called soft} \\
  (p^+, p^-, p_\perp) &\sim (\lambda^2, 1, \lambda) \text{ is called collinear}
  \end{align*}

- Jets have a collinear momentum, i.e., they have a large momentum component in one light cone direction, but only a small invariant mass
- Integrate out the hard modes and the off-cone components of the collinear modes to find the SCET Lagrangian for collinear fields

$$
\mathcal{L} = \bar{\xi} i\bar{n} \cdot D \frac{\gamma}{2} \xi + \bar{\xi} iD_\perp \frac{1}{in \cdot D} i\bar{\gamma}_\perp \frac{\gamma}{2} \xi + \mathcal{L}_{Y.M.}, \quad iD = i\partial + gA
$$
SCET Modes

- **Soft**: Typical representative of the medium; no leading power collinear-soft interaction in the SCET Lagrangian, but

\[
\begin{array}{c}
\text{c} \\
\downarrow \\
\text{s} \\
\text{c} \\
\end{array} \rightarrow \begin{array}{c}
\text{c} \\
\downarrow \\
\text{s} \\
\text{c} \\
\end{array} 
\]

(only if + components of soft momenta add up to \( \lambda^2 \))

- Other possible modes interacting with a collinear quark \((p^+, p^-, p_\perp) \sim (\lambda^2, \lambda^2, \lambda^2)\) is called **ultrasoft**

Decouple at leading power as proven in Bauer et al. ’01
In-medium Interactions

- The most relevant mode for jet broadening $(p^+, p^-, p_\perp) \sim (\lambda^2, \lambda^2, \lambda)$ is called **Glauber**
  Necessary for consistence in exclusive Drell-Yan with spectator interactions Bauer, Lange, Ovanesyan ’10
  and also for interactions with a medium Idilbi, Majumder ’08
  “longitudinal” **Glauber** $(\lambda^2, \lambda, \lambda)$: Also found to be important Ovanesyan, Vitev ’11
  recently in the context of longitudinal drag Qin, Majumder ’12

- Introduce Glauber field into the SCET Lagrangian as an effective classical field of the medium particles
Glauber extended SCET

- In order to determine the importance of these interactions the **scaling** of the Glauber field itself is relevant

\[ A^\mu(x) = \int d^4 y \, D^{\mu\nu}(x - y) J_\nu(y) \]

- Depends on the gauge used as well as on the source

- **Covariant gauge** and soft source: Longitudinal Glauber are more important with components scaling \( A^{+ \text{cov}} \sim A^{\perp \text{cov}} \sim \lambda^2 \)

  Idilbi, Majumder '08; Ovanesyan, Vitev '11

- At leading order in the power counting the Lagrangian for the interaction of collinear particles with Glauber gluons is just

\[ \mathcal{L} = \bar{\xi} i \vec{n} \cdot D \frac{\bar{n}}{2} \xi, \quad iD = i\partial + gA \]
Calculation of \( P(k_\perp) \)

- Determine the probability \( P(k_\perp) \) by calculating the amplitude for the interaction of the collinear quark with gluons from the medium
- First attempt: Use SCET\(_G\) in **covariant gauge**

  D’Eramo, Liu, Rajagopal ’10

- Use optical theorem to determine scattering amplitude

![Diagram of quark scattering](image)

- Initially on-shell quark scattering on an arbitrary number of medium particles via Glauber exchange
- Type of source relevant for eikonalization
$P(k_\perp)$ in covariant gauge

- Result is the Fourier transform of the medium averaged expectation value of two Wilson lines

$$P(k_\perp) = \int d^2x_\perp e^{ik_\perp \cdot x_\perp} \frac{1}{N_c} \left\langle \text{Tr} \left[ W_F^{\dagger}[0, x_\perp] W_F[0, 0] \right] \right\rangle$$

$$W_F[y^+, y_\perp] = \mathcal{P} \left\{ \exp \left[ ig \int_{-\infty}^{\infty} dy^- A^+(y^+, y^-, y_\perp) \right] \right\}$$

- Agrees with Baier et al. '97; Zakharov '96; Casalderrey-Solana, Salgado '07 except time ordering

$$\begin{array}{c}
(0, -\infty, x_\perp) \quad \text{-----} \quad (0, \infty, x_\perp) \\
(0, -\infty, 0) \quad \text{-----} \quad (0, \infty, 0)
\end{array}$$

- Not gauge invariant ($W_F = 1$ in light-cone gauge $A^+ = 0$)
**Changes in arbitrary gauge**

**Goal**

Want to show that SCET\(_G\) is complete and find a gauge invariant expression of \(P(k_\perp)\) for further calculations (e.g. lattice)

- In singular gauges, such as light-cone gauge the scaling of the Glauber field is **different**
  
  Idilbi, Majumder ’08; Ovanesyan, Vitev ’11
  
  \[ A_{\perp}^{\text{cov}} \ll A_{\perp}^{\text{lcg}} \]

- This can be traced back to the factor \(k_\perp/[k^+]\) appearing in the Fourier transform of the gluon propagator in light-cone gauge (the square brackets indicate an appropriate regularization for he light-cone singularity)

- Additional leading power interaction term in the Lagrangian becomes relevant

\[
\bar{\xi} iD_\perp \frac{1}{Q} iD_\perp \frac{\eta}{2} \xi
\]

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\( \hat{\eta} \) and EFT

MLL Kolloquium, June 2014
Changes in arbitrary gauge

- Additional vertices for collinear-Glauber interaction

\[
\begin{array}{cc}
  & A_\perp & A_\perp \\
\end{array}
\]

- Summing over any number of gluon interactions, we find in light-cone gauge

\[
P(k_\perp) = \frac{1}{N_c} \int d^2 x_\perp \ e^{i k_\perp \cdot x_\perp} \langle \text{Tr} \left[ T^\dagger(0, -\infty, x_\perp) \ T(0, \infty, x_\perp) \ T^\dagger(0, \infty, 0) \ T(0, -\infty, 0) \right] \rangle
\]

- with

\[
T(x_+, \pm \infty, x_\perp) = \mathcal{P} e^{-ig \int_{-\infty}^{0} ds \ l_\perp \cdot A_\perp(x_+, \pm \infty, x_\perp + l_\perp s)}
\]

the transverse Wilson line
Wilson lines in the perpendicular plane at $\pm\infty$ for light-cone gauge
Combining the results with the ones in covariant gauge we find

\[
P(k_{\perp}) = \frac{1}{N_c} \int d^2x_{\perp} e^{ik_{\perp} \cdot x_{\perp}} \langle Tr\left[ T(0, -\infty, x_{\perp}) W_F[0, x_{\perp}] T(0, \infty, x_{\perp}) T^\dagger(0, \infty, 0) W_F[0, 0] T(0, -\infty, 0) \right] \rangle
\]

Note that for certain regularizations of the light-cone singularity of the gluon propagator, the Glauber field might vanish at either $+\infty^-$ or $-\infty^-$ even in light-cone gauge

Liang, Wang, Zhou ’08
Results combined

(0, −∞, x⊥) → (0, ∞, x⊥)

(0, −∞, 0) → (0, ∞, 0)

(0, −∞, −∞ l⊥) → (0, ∞, −∞ l⊥)
Results combined

- Transverse Wilson lines combine to

\[(0, -\infty, x_{\perp})(0, -\infty, 0) \rightarrow (0, \infty, x_{\perp})(0, \infty, 0)\]

- Fields on the lower line are time ordered, the ones on the upper line anti-time ordered
  
  \( \rightarrow \) Use **Schwinger-Keldysh contour** in path integral formalism

Next step

Field theoretic definition complete. Can further information be extracted through perturbative calculations?
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Thermal field theories

- In order to actually perform calculations an additional set of effective field theories may be applied.

- Compare the **thermal average** of an equal time operator
  \[ \langle A \rangle = \frac{1}{Z} \text{Tr} \left( e^{-H\beta} A \right) \]
  to the path integral representation of expectation value
  \[ \langle \phi | e^{-iHT} | \phi \rangle = \int \mathcal{D}\phi \exp \left( i \int_0^T d^4x \mathcal{L} \right) \]

- The **imaginary time formalism** relates the thermal average to a path integral along an imaginary time coordinate.

- Wave functions are periodic in imaginary time \( \rightarrow \) Fourier series
  \[ \phi(\vec{x},\tau) \sim \sum_n \int \frac{d^3p}{(2\pi)^3} e^{i(\vec{p}\vec{x}+\omega_n\tau)} \phi(\vec{p},\omega_n) \]
  with \( \omega_n = 2\pi n T \) the Matsubara frequency.
Electrostatic QCD

Correspondingly the propagator decomposes into

$$G(t = 0, \vec{x}) = T \sum_n \int \frac{d^3 p}{(2\pi)^3} e^{i\vec{p} \cdot \vec{x}} \, G_E(\omega_n, \vec{p})$$

$$G_E(\omega_n, \vec{p}) \sim \frac{1}{\omega_n^2 + \vec{p}^2} \quad \text{Euclidean propagator}$$

Integrate out the Matsubara modes $n > 0 \ (\sim \pi T)$ and redefine $A^\mu \rightarrow \sqrt{T} A^\mu \rightarrow \text{electrostatic QCD (EQCD)}$ Braaten '95

3D Euclidean Yang-Mills coupled to massive scalar $A^0$ with mass $m_E \sim gT$ and coupling $g_E \sim g^2 T$

Applicable for $P(k_\perp)$ if $k_\perp \sim gT$

EQCD Lagrangian

$$\mathcal{L}_{\text{EQCD}} = \frac{1}{4} G_{ij}^a G_{ij}^a + \frac{1}{2} (D_i A_0)^2 + \frac{1}{2} m_E^2 A_0^2$$
Issues and solutions

- Fields in $P(k_\perp)$ are path ordered and have to be calculated on a Schwinger-Keldysh contour

![Schwinger-Keldysh contour diagram]

but symmetric propagator (connecting fields on the same part of the contour) is enhanced by $T/k^0 \rightarrow$ use time ordering

- Need thermal average of light-like Wilson lines, but imaginary time formalism only applicable to equal time correlators

$\rightarrow$ **Deform contour** to be slightly space-like ($\nu = 1 + \epsilon$), then boost to equal time and use imaginary time formalism Caron-Huot ’08
Relation to the Wilson loop

- So far have argued that $P(k_\perp)$ for $k_\perp \sim gT$ is related to the expectation value of a “time-ordered set of Wilson lines of a 3D-Yang-Mills theory (plus the $A^0$ field)
- This object is very similar to the expectation value of the 3D static Wilson loop
- The probability $P(k_\perp)$ can therefore be related to the static energy in 3D Yang-Mills theory

$$P(k_\perp)_{\text{F.T.}} \sim \frac{1}{N_c} \langle \text{Tr} \begin{array} \hline & \downarrow & \\ \downarrow & \circ \end{array} \rangle = e^{-h_s(x_\perp)L} \quad \text{for } L \to \infty$$

Caron-Huot '08

- Advantage: A lot of calculations have already been performed for the static energy
- When also integrating out the scale $m_E \sim gT$ (i.e. the field $A^0$) one arrives at magnetostatic QCD (MQCD) Braaten '95
- The probability $P(k_\perp)$ for $k_\perp \sim g^2T$ is then exactly related to the static energy and computed on the lattice Luescher, Weisz '02
Perturbative and lattice calculations

- Use this analogy to calculate contributions to \( \hat{q} \). What is known?
- Interference of loop and power expansion
  
  \[
  \begin{align*}
  \text{LO} & \sim g^4 T^3 \quad \text{Arnold, Xiao '08 (from } k_\perp \sim T \text{ and } k_\perp \sim gT) \\
  \text{NLO} & \sim g^5 T^3 \quad \text{Caron-Huot '08 (from } k_\perp \sim gT \text{ with loops)} \\
  \text{NNLO} & \sim g^6 T^3 \quad \text{from } k_\perp \sim g^2 T \text{ using lattice data for the static potential} \\
  \end{align*}
  \]

  Laine '12; Mi.B., Brambilla, Escobedo, Vairo '12

  gauge invariant result was needed!

- But computation of NNLO is still missing contributions from
  \[ k_\perp \gg g^2 T \]
NNLO contribution

- Calculations corresponding to the contributions for $k_{\perp} \sim gT$ are available for the static potential in 3D QCD
  
  Schroeder '99; Pineda, Stahlhofen '10

- There it was shown, that a naive computation yields a infrared divergent result
  
  It was demonstrated, that a matching onto a low energy effective theory (pNRQCD) regulates this divergence
  
  \[ h_s(x_{\perp}) = V_s(x_{\perp}, \mu) + \delta h_s(x_{\perp}, \mu) \]

- We will use the analogy to the jet quenching case to determine the logarithmic contributions at NNLO to $P(k_{\perp})$
  
  The role of pNRQCD will be played by MQCD
  
  \[ \rightarrow \text{work in progress} \]
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Conclusions

- Jet quenching is an useful phenomenon that yields insights into the properties of a quark-gluon plasma
- Due to the appearance of several different scales in practical computations, a series of effective field theories is introduced
- $\text{SCET}_G$ is a suitable theory to give a gauge invariant field theoretical definition of the jet quenching parameter
- The jet quenching parameter $\hat{q}$ can be expressed as the medium average of two longitudinal and four transverse Wilson lines
- In the appropriate momentum region this medium average can be computed in $\text{EQCD}$
- There is an analogy to the static Wilson loop in 3D Yang-Mills
- By matching onto $\text{MQCD}$ it is possible to use known results to derive the NNLL contributions to $P(k_\perp)$
Thank you for your attention!
Bonus Slides
Scaling of the Glauber field

Consider the form of the effective Glauber field

\[ A^\mu(x) = \int d^4y \ D^\mu\nu_G(x - y) f_\nu(y) \]

\[ D^\mu\nu(x - y) = \int \frac{d^4k}{(2\pi)^4} \frac{-i}{k^2 + i\epsilon} \left( g^\mu\nu - \frac{k^\mu \bar{n}^\nu + k^\nu \bar{n}^\mu}{[k^+]} \right) e^{-ik(x-y)} \]

Source \( f_\nu \) only knows about the soft scale \( \sim \lambda^3 \)
The gauge field at light-cone infinity

- The gluon field may be decomposed

\[ A^i_\perp(x^+, x^-, x_\perp) = A^{\text{cov}, i}_\perp(x^+, x^-, x_\perp) + \theta(x^-)A^i_\perp(x^+, \infty, x_\perp) + \theta(-x^-)A^i_\perp(x^+, -\infty, x_\perp) \]

where \( A^{\text{cov}, i}_\perp \) corresponds to the non-singular part of the propagator and vanishes at \( \pm \infty^- \) and where the leading power comes from the terms at \( \infty^- \)

Echevarria, Idilbi, Scimemi ’11

- For \( x^- \to \infty \) the field strength must vanish

\[ A_\perp(x^+, \infty, x_\perp) \text{ is a pure gauge} \]

\[ A_\perp(x^+, \infty, x_\perp) = \nabla_\perp \phi(x^+, \infty, x_\perp) \]

\[ \phi(x^+, \infty, x_\perp) = - \int_{-\infty}^{0} ds \, l_\perp \cdot A_\perp(x^+, \infty, x_\perp + l_\perp s) \]

Belitsky, Ji, Yuan ’02
Calculation

- Define the (amputated) diagram with $n$ gluon interactions

$$G_n(k) = \quad \text{Diagram with } n \text{ gluon interactions}$$

- We can calculate this in a recursive fashion
Calculation

- Decompose into fields at $\pm \infty$

$$G_n(k^-, k_\perp) = \sum_{j=0}^{n} \int \frac{d^4q}{(2\pi)^4} \ G_{n-j}(k^-, k_\perp, q) \ \frac{iQ \hat{n}}{2Q q^+ - q_\perp^2 + i\epsilon} \ G_j^-(q)$$

where $G^\pm$ contains only the gluon at $\pm \infty$

- The recursive definition of $G^-$ is then

$$G_n^-(q) = \int \frac{d^4q'}{(2\pi)^4} \ G_{n-1}^-(q') \quad q' \rightarrow q$$

$$+ \int \frac{d^4q''}{(2\pi)^4} \ G_{n-2}^-(q'') \quad q'' \rightarrow q$$

- and $G_n^+$ correspondingly
Collinear-soft interactions

\[ p_c^- (k_s^+ + k_s'^+) + p_{c\perp} (k_s\perp + k_s'^\perp) -- O(\lambda^3) \]

- SCET operator
  \[ \bar{\xi} \gamma^\mu gA_s^+ S\xi \]

- S soft Wilson line
On the lattice

- $\hat{q}$ in terms of the static potential

$$\hat{q} |_{g^2 T} = - (q^*)^2 \int_0^\infty d\lambda \lambda^3 J_0(\lambda) \int_\lambda^\infty \frac{dz}{z^3} V \left( \frac{z}{q^*} \right)$$

Luescher, Weisz '02

$$V(r) = \frac{1}{r_0} \left( a \frac{r}{r_0} - b \frac{r_0}{r} + \ldots \right)$$
Perturbative results

- **NLO result** Arnold, Xiao '08; Caron-Huot '08

\[
\hat{q}(q_{\text{max}}) = \frac{2g^4 T^3}{3\pi} \left[ \frac{3}{2} \log \left( \frac{T}{m_D} \right) + \frac{7\zeta(3)}{4\zeta(2)} \log \left( \frac{q_{\text{max}}}{T} \right) - 0.105283 \right] \\
+ \frac{g^4 T^3}{8\pi^2} \frac{m_D}{T} \left( 3\pi^2 + 10 - 4 \log(2) \right) \\
\approx (2.08 + 5.26) \left( \frac{T}{\text{GeV}} \right)^3 \text{GeV}^2/\text{fm}
\]