Heavy Hybrids in pNRQCD

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In the last decade many new unexpected states have been found close or above threshold.

The states that do not fit Quarkonium potential models are called Exotics and labeled Xs, Ys and Zs.

This states are candidates for non traditional hadronic states, including four constituent quark or an excited gluon constituent.

There is an ongoing experimental effort to study normal and Exotic quarkonium: new states, production mechanisms, decays and transitions, precision and high statistics data.

BaBar, Belle2, BESIII, LHCb and Panda (under construction).

Voloshin
# Quarkonium Hybrids

## What are quarkonium Hybrids?

- A quarkonium hybrid consists of $Q$, $\bar{Q}$ in a color octet configuration and a gluonic excitation $g$.

### Born-Oppenheimer Hybrids:

The heavy quarks are nearly static, and the gluons adapt nearly instantaneously.

## Born-Oppenheimer approximation Heavy Hybrids

- The gluonic static energies can be defined in NRQCD and computed on the lattice or, in the short range, using pNRQCD.
- The hybrid state energy levels are obtained solving the Schrödinger equation with $H_{\text{kin}} + E_g$.
- $H_{\text{kin}}$ acts on the gluon wave functions. Additional approximations are needed, because the gluonic wave functions are not available.
- The mixing terms have to be taken into account because the static energies are degenerate at short distances.

Pioneered by Juge, Kuti, Morningstar 1999
Symmetries of the static system

Static states classified by symmetry group $D_{\infty h}$

Representations labeled $\Lambda^\sigma_{\eta}$

- $\Lambda$ rotational quantum number
  $|\hat{n} \cdot \mathbf{K}| = 0, 1, 2 \ldots$ corresponds to
  $\Lambda = \Sigma, \Pi, \Delta \ldots$

- $\eta$ eigenvalue of $CP$:
  $g \hat{=} + 1$ (gerade), $u \hat{=} - 1$ (ungerade)

- $\sigma$ eigenvalue of reflections

- $\sigma$ label only displayed on $\Sigma$ states
  (others are degenerate)

- The static energies correspond to the irreducible representations of $D_{\infty h}$.
- In general it can be more than one state for each irreducible representations of
  $D_{\infty h}$, usually denoted by primes, e.g. $\Pi_u, \Pi'_u, \Pi''_u \ldots$
Lattice data on hybrid static energies

- $\Sigma_{g}^{+}$ is the ground state potential that generates the standard quarkonium states.
- The rest of the static energies correspond to excited gluonic states that generate hybrids.
- The two lowest hybrid static energies are $\Pi_{u}$ and $\Sigma_{u}^{-}$, they are nearly degenerate at short distances.
- The static energies have been computed in quenched lattice QCD, the most recent data by Juge, Kuti, Morningstar, 2002 and Bali and Pineda 2003.
- Quenched and unquenched calculations for $\Sigma_{g}^{+}$ and $\Pi_{u}$ were compared in Bali et al 2000 and good agreement was found below string breaking distance.
Radial wave functions and Schrödinger equation

The hybrid masses are obtained by solving the Schrödinger equation with \( H_{\text{kin}} + E_{n\Lambda \eta} \). Let us define Hybrid state as:

\[
\left| N; j, m, s, l; n, \eta; \epsilon \right\rangle = \sum_{\Lambda} \int dr \left| j, m, s, l; n, \Lambda, \eta; \epsilon \right\rangle \frac{\psi^\Lambda_N(r)}{r}.
\]

\( H_{\text{kin}} = -\frac{\partial_r^2}{2\mu} + \frac{L_{QQ}^2}{2\mu r^2} \) acts in principle on both the gluonic and heavy quark parts.

The radial derivative acting on the gluonic state can be ignored.

Angular derivative

In spherical coordinates \( \mathbf{K} = K_\theta \hat{\theta} + K_\phi \hat{\phi} + K_n \hat{n} \), and the rising and lowering operators \( K_\pm = K_\theta \pm iK_\phi \).

\[
L_{QQ}^2 = (L - \mathbf{K})^2 = L^2 - 2L \cdot \mathbf{K} + \mathbf{K}^2 = L^2 - 2K_n^2 + \mathbf{K}^2 - L_- K_+ - L_+ K_-,
\]

- Acting on our \( J^{PC} \) eigenstates the first two terms give \( l(l+1) \) and \(-2\Lambda^2\).
- \( \mathbf{K}^2 - L_- K_+ - L_+ K_- \) is not directly determined since we do not have the gluonic wave functions.
- \( L_- K_+ + L_+ K_- \) mixes different channels.
To determine the last two terms we use:

1. \( \frac{L^2_{QQ}}{2\mu r^2} \) is most important in the short range.

2. The static symmetry group \( D_{\infty h} \) is extended to \( O(3) \times C \) in the limit \( r \to 0 \).
   ⇒ approximate \( K^2 - L_- K_+ - L_+ K_- \) for its short range behavior.

**\( K^2 \)**

- \( \langle K^2 \rangle = k(k + 1) \) with \( k \) from the \( K^{PC} \) multiplet corresponding to \( n\Lambda^\sigma_{\eta} \) in the \( r \to 0 \) limit.
- For the three the lowest static energies, \( \Sigma^+_g, \Pi_u \) and \( \Sigma^-_u \), pNRQCD tells us these are \( 0^{++}, 1^{+-} \) and \( 1^{+-} \) respectively.

**\( L_- K_+ + L_+ K_- \)**

- If we ignore the mixing terms, for \( \Lambda > 0 \), there are two degenerate states with opposite parity. If we include them the degeneracy is lifted and we obtain an effect called \( \Lambda \)-doubling.
- The mixing involves gluonic static energies with \( \lambda \) and \( \lambda \pm 1 \).
- We will only consider mixing between static energies that are nearly degenerate.
- We have considered the mixing through **coupled Schrödinger equations**.
Gluonic static energies in pNRQCD

*EFT of QCD for Quarkonium*

- Quarkonium systems are non–relativistic bound states.
- **Multiscale system:** \( m \gg p_Q \gg E_b \), and \( \Lambda_{QCD} \). \( m \) is the heavy–quark mass.
- We can exploit the scale hierarchies by building an **Effective Field Theory** (EFT).

*pNRQCD for Hybrid static energies*

- The short range \( p_Q \sim 1/r \gg \Lambda_{QCD} \) behavior of the static energies can be studied in weakly-coupled pNRQCD.
- In this region pNRQCD is obtained integrating out \( p_Q \) (perturbative) and \( \Lambda_{QCD} \) nonperturbative.
The Gluelumps are the adjoint sources in the presence of a gluonic field

\[ H(R, r, t) = H^a(R, t) \mathcal{O}^{a\dagger}(R, r, t) , \]

Gluonic excitation operators up to dim 3

<table>
<thead>
<tr>
<th>( \Lambda_{\eta}^\sigma )</th>
<th>( K^{PC} )</th>
<th>( H^a )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Sigma_u^- )</td>
<td>1+-</td>
<td>( r \cdot B, r \cdot (D \times E) )</td>
</tr>
<tr>
<td>( \Pi_u )</td>
<td>1+-</td>
<td>( r \times B, r \times (D \times E) )</td>
</tr>
<tr>
<td>( \Sigma_{g'}^+ )</td>
<td>1--</td>
<td>( r \cdot E, r \cdot (D \times B) )</td>
</tr>
<tr>
<td>( \Pi_g )</td>
<td>1--</td>
<td>( r \times E, r \times (D \times B) )</td>
</tr>
<tr>
<td>( \Sigma_g^- )</td>
<td>2--</td>
<td>( (r \cdot D)(r \cdot B) )</td>
</tr>
<tr>
<td>( \Pi_g' )</td>
<td>2--</td>
<td>( r \times ((r \cdot D)B + D(r \cdot B)) )</td>
</tr>
<tr>
<td>( \Delta_g )</td>
<td>2--</td>
<td>( (r \times D)^i(r \cdot B)^j + (r \times D)^j(r \cdot B)^i )</td>
</tr>
<tr>
<td>( \Sigma_u^+ )</td>
<td>2+-</td>
<td>( (r \cdot D)(r \cdot E) )</td>
</tr>
<tr>
<td>( \Pi_u' )</td>
<td>2+-</td>
<td>( r \times ((r \cdot D)E + D(r \cdot E)) )</td>
</tr>
<tr>
<td>( \Delta_u )</td>
<td>2+-</td>
<td>( (r \times D)^i(r \times E)^j + (r \times D)^j(r \times E)^i )</td>
</tr>
</tbody>
</table>

We can see that in the short distance limit the \( \Pi_u - \Sigma_u^-, \Pi_g - \Sigma_{g'}^+, \Delta_g - \Sigma_g^- - \Pi_g' \) and \( \Delta_u - \Pi_u' - \Sigma_u^+ \) multiplets must be degenerate.
Hybrid Static energies

- The hybrid static energy spectrum reads

\[ E_H = 2m + V_H , \]

with

\[ V_H = \lim_{T \to \infty} \frac{i}{T} \log \left< H^a(T/2) O^a(T/2) H^b(-T/2) O^b(-T/2) \right> . \]

- Up to next-to-leading order in the multipole expansion.

\[ V_H = V_o + \Lambda_H + b_H r^2 , \]

- \( V_o(r) \) is the octet potential, which can be computed in perturbation theory.

- \( \Lambda_H \) corresponds to the gluelump mass.

\[ \Lambda_H = \lim_{T \to \infty} \frac{i}{T} \log \left< H^a(T/2) \phi_{ab}^{adj}(T/2, -T/2) H^b(-T/2) \right> , \]

where \( \phi_{ab}^{adj}(T/2, -T/2) \) is a Wilson line.

- We work in the Renormalon Subtracted scheme which improves the convergence of the octet potential.
**Λ<sub>H</sub>**

- It is a non-perturbative quantity.
- It depends on the particular operator $H^a$, however it is the same for operators corresponding to different projections of the same gluonic operators.
- The gluelump masses have been determined in the lattice. Foster et al. 1999; Bali, Pineda 2004; Marsh Lewis 2014
- At the subtraction scale $\nu_f = 1$ GeV: $\Lambda_{1^+}^{RS} = 0.87(15)$ GeV.

**b<sub>H</sub>**

- It is a non-perturbative quantity.
- Proportional to $r^2$ due to rotational invariance and the multipole expansion.
- We are going to fix it through a fit to the static energies lattice data.
- Breaks the degeneracy of the potentials.
Static Hybrid potentials

\[ \Sigma_u \]

\[ \Pi_u \]

Lattice data: Bali, Pineda 2004; Juge, Kuti, Morningstar 2003, dashed line \( V^{(0.5)} \), solid line \( V^{(0.25)} \)

\( V^{(0.5)} \)

Lattice data fitted for the \( r = 0 - 0.5 \) fm range, \( b^{(0.5)}_\Sigma = 1.112 \text{ GeV/fm}^2 \), \( b^{(0.5)}_\Pi = 0.110 \text{ GeV/fm}^2 \).

\( c_{BP} = 0.105 \text{ GeV} \), \( c_{KJM} = -0.471 \text{ GeV} \),

\( V^{(0.25)} \)

1. \( r \leq 0.25 \) fm: pNRQCD potential, \( b^{(0.25)}_\Sigma = 1.246 \text{ GeV/fm}^2 \), \( b^{(0.25)}_\Pi = 0.000 \text{ GeV/fm}^2 \).
2. \( r > 0.25 \) fm: phenomenological potential, \( V'(r) = \frac{a_1}{r} + \sqrt{a_2 r^2 + a_3} + a_4 \).
Hybrid state masses from $V^{(0.25)}$

Solving the coupled Schrödinger equations we obtain

<table>
<thead>
<tr>
<th>$m_H$</th>
<th>$\langle 1/r \rangle$</th>
<th>$E_{\text{kin}}$</th>
<th>$P_\Pi$</th>
<th>$m_H$</th>
<th>$\langle 1/r \rangle$</th>
<th>$E_{\text{kin}}$</th>
<th>$P_\Pi$</th>
<th>$m_H$</th>
<th>$\langle 1/r \rangle$</th>
<th>$E_{\text{kin}}$</th>
<th>$P_\Pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_1$</td>
<td>4.15</td>
<td>0.42</td>
<td>0.16</td>
<td>0.82</td>
<td>7.48</td>
<td>0.46</td>
<td>0.13</td>
<td>0.83</td>
<td>10.79</td>
<td>0.53</td>
<td>0.09</td>
</tr>
<tr>
<td>$H'_1$</td>
<td>4.51</td>
<td>0.34</td>
<td>0.34</td>
<td>0.87</td>
<td>7.76</td>
<td>0.38</td>
<td>0.27</td>
<td>0.87</td>
<td>10.98</td>
<td>0.47</td>
<td>0.19</td>
</tr>
<tr>
<td>$H_2$</td>
<td>4.28</td>
<td>0.28</td>
<td>0.24</td>
<td>1.00</td>
<td>7.58</td>
<td>0.31</td>
<td>0.19</td>
<td>1.00</td>
<td>10.84</td>
<td>0.37</td>
<td>0.13</td>
</tr>
<tr>
<td>$H'_2$</td>
<td>4.67</td>
<td>0.25</td>
<td>0.42</td>
<td>1.00</td>
<td>7.89</td>
<td>0.28</td>
<td>0.34</td>
<td>1.00</td>
<td>11.06</td>
<td>0.34</td>
<td>0.23</td>
</tr>
<tr>
<td>$H_3$</td>
<td>4.59</td>
<td>0.32</td>
<td>0.32</td>
<td>0.00</td>
<td>7.85</td>
<td>0.37</td>
<td>0.27</td>
<td>0.00</td>
<td>11.06</td>
<td>0.46</td>
<td>0.19</td>
</tr>
<tr>
<td>$H_4$</td>
<td>4.37</td>
<td>0.28</td>
<td>0.27</td>
<td>0.83</td>
<td>7.65</td>
<td>0.31</td>
<td>0.22</td>
<td>0.84</td>
<td>10.90</td>
<td>0.37</td>
<td>0.15</td>
</tr>
<tr>
<td>$H_5$</td>
<td>4.48</td>
<td>0.23</td>
<td>0.33</td>
<td>1.00</td>
<td>7.73</td>
<td>0.25</td>
<td>0.27</td>
<td>1.00</td>
<td>10.95</td>
<td>0.30</td>
<td>0.18</td>
</tr>
<tr>
<td>$H_6$</td>
<td>4.57</td>
<td>0.22</td>
<td>0.37</td>
<td>0.85</td>
<td>7.82</td>
<td>0.25</td>
<td>0.30</td>
<td>0.87</td>
<td>11.01</td>
<td>0.30</td>
<td>0.20</td>
</tr>
<tr>
<td>$H_7$</td>
<td>4.67</td>
<td>0.19</td>
<td>0.43</td>
<td>1.00</td>
<td>7.89</td>
<td>0.22</td>
<td>0.35</td>
<td>1.00</td>
<td>11.05</td>
<td>0.26</td>
<td>0.24</td>
</tr>
</tbody>
</table>

Consistency test:

1. The multipole expansion requires
   $\langle 1/r \rangle > E_{\text{kin}}$.

Conclusion:

- $V^{(0.25)}$ yields more consistent results.
- As expected the Born–Oppenheimer program works better in bottomonium than charmonium

Spin symmetry multiplets

<table>
<thead>
<tr>
<th>$H_1$</th>
<th>${1--,(0,1,2)^{+-}}$</th>
<th>$\Sigma_u^-, \Pi_u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_2$</td>
<td>${1++, (0,1,2)^{+-}}$</td>
<td>$\Pi_u$</td>
</tr>
<tr>
<td>$H_3$</td>
<td>${0++, 1^{+-}}$</td>
<td>$\Sigma_u^-$</td>
</tr>
<tr>
<td>$H_4$</td>
<td>${2++, (1,2,3)^{+-}}$</td>
<td>$\Sigma_u^-, \Pi_u$</td>
</tr>
<tr>
<td>$H_5$</td>
<td>${2--, (1,2,3)^{-+}}$</td>
<td>$\Pi_u$</td>
</tr>
<tr>
<td>$H_6$</td>
<td>${3--, (2,3,4)^{-+}}$</td>
<td>$\Sigma_u^-, \Pi_u$</td>
</tr>
<tr>
<td>$H_7$</td>
<td>${3++, (2,3,4)^{+-}}$</td>
<td>$\Pi_u$</td>
</tr>
</tbody>
</table>
In Braaten et al 2014 a similar procedure was followed to obtain the hybrid masses.

No Λ–doubling effect mixing terms were included, and phenomenological potentials fitting the lattice data.

We can compare the results to estimate the size of the Λ–doubling effect.

Charmonium sector

The mixing lowers the mass of the $H_1(H_4)$ multiplet with respect to $H_2(H_4)$.
Identification with experimental states

Most of the candidates have $1^{--}$ or $0^{++}/2^{++}$ since the main observation channels are production by $e^+e^-$ or $\gamma\gamma$ annihilation respectively.

- **Charmonium states (Belle, CDF, BESIII, Babar):**

  ![Charmonium States Diagram]

- **Bottomonium states:** $Y_b(10890)[1^{--}]$, $m = 10.8884 \pm 3.0$ (Belle). Possible $H_1$ candidate, $m_{H_1} = 10.79 \pm 0.15$.

  **However**, except for $Y(4220)$, all other candidates observed decay modes violate Heavy Quark Spin Symmetry.
Comparison with direct lattice computations

Charmonium sector

- Calculations done by the Hadron Spectrum Collaboration using unquenched lattice QCD with a pion mass of 400 MeV. Liu et al. 2012
- They worked in the constituent gluon picture, which consider the multiplets $H_2$, $H_3$, $H_4$ as part of the same multiplet.
- Their results are given with the $\eta_c$ mass subtracted.

![Graph showing mass differences between multiplets](image)

Error bands take into account the uncertainty on the gluelump mass $\pm 0.15$ GeV

<table>
<thead>
<tr>
<th>Split (GeV)</th>
<th>Liu</th>
<th>$V^{(0.25)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta m_{H_2-H_1}$</td>
<td>0.10</td>
<td>0.13</td>
</tr>
<tr>
<td>$\delta m_{H_4-H_1}$</td>
<td>0.24</td>
<td>0.22</td>
</tr>
<tr>
<td>$\delta m_{H_4-H_2}$</td>
<td>0.13</td>
<td>0.09</td>
</tr>
<tr>
<td>$\delta m_{H_3-H_1}$</td>
<td>0.20</td>
<td>0.44</td>
</tr>
<tr>
<td>$\delta m_{H_3-H_2}$</td>
<td>0.09</td>
<td>0.31</td>
</tr>
</tbody>
</table>

- Our masses are $0.1 - 0.14$ GeV lower except the for the $H_3$ multiplet, which is the only one dominated by $\Sigma^u$.
- Good agreement with the mass gaps between multiplets, in particular the $\Lambda$-doubling effect ($\delta m_{H_2-H_1}$).
Conclusions

- We have computed the heavy hybrid masses using a QCD analog of the Born-Oppenheimer approximation including the Λ–doubling terms by using coupled Schröringer equations.
- The static energies have been obtained combining pNRQCD for short distances and lattice data for long distances.
- A large set of masses for spin symmetry multiplets for $c\bar{c}$, $b\bar{c}$ and $b\bar{b}$ has been obtained.
- Λ–doubling effect lowers the mass of the multiplets generated by a mix of static energies, the same pattern is observed in direct lattice calculations and QCD sum rules.
- Mass gaps between multiplets in good agreement with direct lattice computations, but the absolute values are shifted.
- Several experimental candidates for Charmonium hybrids belonging to the $H_1$, $H_2$, $H_4$ and $H'_1$ multiplets.
- One experimental candidate to the bottomonium $H_1$ multiplet.
Thank you for your attention
The mixing lowers the mass of the $H_1(H_4)$ multiplet with respect to $H_2(H_4)$. 

$\Lambda$-doubling effect
Comparison with direct lattice computations

Bottomonium sector

- Calculations done by Juge, Kuti, Morningstar 1999 and Liao, Manke 2002 using quenched lattice QCD.
- Juge, Kuti, Morningstar 1999 included no spin or relativistic effects.
- Liao, Manke 2002 calculations are fully relativistic.

Error bands take into account the uncertainty on the gluelump mass ±0.15 GeV

<table>
<thead>
<tr>
<th>Split (GeV)</th>
<th>JKM</th>
<th>$\sqrt{\mathcal{V}(0.25)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta m_{H_2-H_1}$</td>
<td>0.04</td>
<td>0.05</td>
</tr>
<tr>
<td>$\delta m_{H_3-H_1}$</td>
<td>0.33</td>
<td>0.27</td>
</tr>
<tr>
<td>$\delta m_{H_3-H_2}$</td>
<td>0.30</td>
<td>0.22</td>
</tr>
<tr>
<td>$\delta m_{H'_1-H_1}$</td>
<td>0.42</td>
<td>0.19</td>
</tr>
</tbody>
</table>

- Our masses are 0.15 – 0.25 GeV lower except the for the $H'_1$ multiplet, which is larger by 0.36 GeV.
- Good agreement with the mass gaps between multiplets, in particular the $\Lambda$-doubling effect ($\delta m_{H_2-H_1}$).
Comparison with QCD sum rules

- A recent analysis of QCD sum rules for hybrid operators has been performed by Chen et al. 2013, 2014 for $b\bar{b}$ and $c\bar{c}$ hybrids, and $b\bar{c}$ hybrids respectively.
- Correlation functions and spectral functions were computed up to dimension six condensates which stabilized the mass predictions compared to previous calculations which only included up to dimension 4 condensates.

Charmonium sector Chen et al. 2013

![Graph showing mass (GeV) vs. quantum numbers for $H_1, H_2, H_3, H_4$ multiplets.]

Error bands take into account the uncertainty on the gluelump mass ±0.15 GeV

- The spin average of the $H_1$ multiplet is 0.4 GeV lower than our mass.
- $H_2, H_3$ and $H_4$ multiplets are incomplete.
- Large uncertainties compared to direct lattice calculations.
Comparison with QCD sum rules

Bottomonium sector Chen et al 2013

Error bands take into account the uncertainty on the gluelump mass ±0.15 GeV

- The spin average of the $H_1$ multiplet is 0.98 GeV lower than our mass.
- $H_2$, $H_3$ and $H_4$ multiplets are incomplete.
- Large uncertainties compared to direct lattice calculations.